

OPERATIONS RESEARCH IN THE BASIC DESIGN OF YS-11 TRANSPORT AIRPLANE

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Summary—YS-11 is a 52/60 passenger, twin turboprop transport airplane designed to have the lowest direct operating cost within short stage length up to 600 nautical miles. As the results of our operations research, we estimated a considerable increase of domestic passengers during the coming five to ten years, and we found that we could realize bigger profits by using transport planes with somewhat larger capacity than the expected number of passengers per flight. Thus, we decided to have as many seats as our budget would allow us, namely 52/60 seats, though YS-11 was considered as a modernized replacement of the 24/30 passenger DC-3.

1. POST-WAR JAPANESE AIRCRAFT INDUSTRY

YS-11 is a 52/60 passenger, short range, twin turboprop transport airplane as shown in Fig. 1 which, at present, is being designed and built in Japan. The first flight of its prototype is scheduled to be toward the end of 1961. Although the main purpose of this paper is to give an outline of the operations research of the basic design of YS-11, we should first

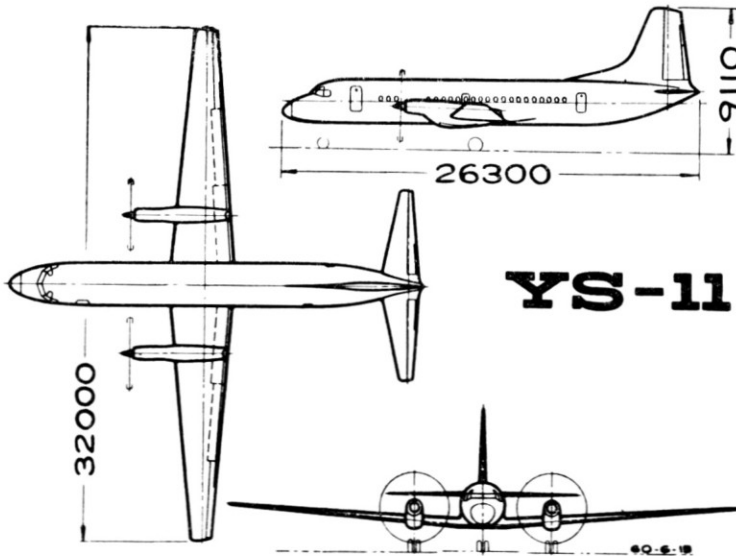


FIG. 1. Three-view drawing of YS-11.

like to explain the situation lying behind recent developments of Japanese aircraft industry.

It was in April 1952, when the Peace Treaty had been ratified, that Japanese aircraft industry was revived after seven years of inactivity. The United States had been engaged in the Korean War and had issued an order to Japanese makers to manufacture various kinds of consumable parts, in order to simplify the massive logistics problems in the Far East. With the revival of aircraft industry, Shin-Meiwa Industry (the former Kawanishi Aircraft Co.) received an order to manufacture jet aircraft drop tanks. Since July 1952, Showa Aircraft Co., Kawasaki Aircraft Co., Shin-Mitsubishi Heavy Industries, Nihon Aircraft Co., and Shin-Meiwa Industry received orders for maintenance, repair and overhaul of U.S. military planes and helicopters, while since 1954, Kawasaki Aircraft Co. received an order to overhaul U.S. military jet engines. Some other companies received orders to repair or overhaul instruments and accessories. Such maintenance and overhaul orders were valuable as a means of acquiring new technique and were beneficial in overcoming economic difficulties confronting Japanese aircraft industry after its seven years of inactivity.

In 1954, the Japanese Self Defense Forces were established. This acted as a strong support for Japanese aircraft industry. In the beginning, aircraft used by Air, Maritime and Ground Forces were provided from the United States under the mutual aid programme. But, presently, they began to be manufactured in Japan under licence. In January 1956, Kawasaki Aircraft Co. succeeded in manufacturing the first of T-33 jet trainers. At present, F-86F jet fighters, P2V-7 Neptune patrols, T-34 Mentor trainers, L-19 Liaisons, S-55 and Bell 47 helicopters are manufactured under licence for the Self Defense Forces. The output of aircraft industry since its revival up to the present is as follows:

	Number of aircraft produced	Number of employees in the aircraft and parts industry	Total sales of aircraft and parts, including maintenance and overhaul (thousand dollars)
1952	1	959	115
1953	9	3259	7170
1954	36	5321	6690
1955	68	7677	8330
1956	93	14,287	11,790
1957	227	15,426	27,940
1958	158	16,676	52,300
1959	145	16,000	51,450

Since 1957, Japanese aircraft industry has entered its third development stage. For the first time since the war, Fuji Heavy Industries had completed the first post-war jet trainer TIF-2, which was originally designed in Japan, while the newly established TADA (Transport Aircraft Development Association) undertook the basic design of a short-range transport plane. Since Japanese aircraft designers and aircraft industry were totally inexperienced in designing aircraft with turbine engines, the designing of a jet trainer and a turboprop short-range transport had been considered most appropriate objectives.

Even then, to undertake the manufacturing of a 20-ton class transport aircraft was a problem of great difficulty for one single company. Thus, all the existing airframe makers of Japan, namely, Shin-Mitsubishi, Kawasaki, Fuji, Shin-Meiwa, Nihon and Showa, together with several engine

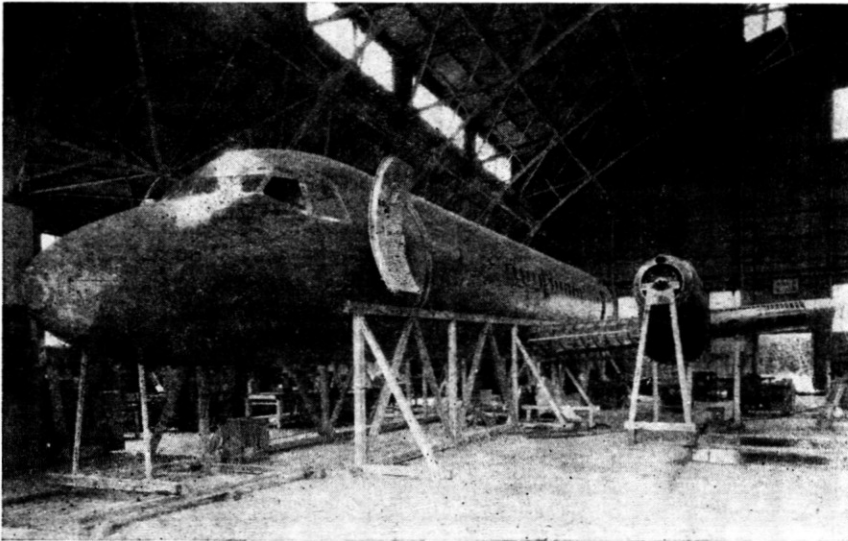


FIG. 2. Mock-up of YS-11.

and instrument makers had organized the TADA. The design staff consisted of engineers provided from each of the above mentioned companies. Being financed with \$425,000 of government subsidy and \$640,000 invested by the members, basic study for short range transport aircraft and basic design of the YS-11 were conducted from 1957 to 1958. In December 1958, the mock-up of YS-11 shown in Fig. 2 was completed.

In June 1959, NAMCO (Nippon Aircraft Manufacturing Co.) was established for the purpose of manufacture and test of YS-11. The basic design of YS-11 made by TADA was handed over to the new company.

As of 1960, the capital of this company is 5.1 million dollars (57% government investment and 43% private investment), while by the end of 1963, 11.6 million dollars of capital is expected.

The design staff was mainly provided by the above mentioned six airframe makers. The manufacturing of the airframe is to be allotted to the same six companies as follows:

Shin-Mitsubishi	front- and center-fuselage, final assembly
Kawasaki	main wing, nacelles
Shin-Meiwa	aft fuselage
Fuji	tail unit
Nihon	flaps and ailerons
Showa	honeycomb parts

Manufacturing of jigs and parts started since the summer of 1960. The RDa-10/1 engines which are to be installed on the prototype have already been ordered from Rolls-Royce Co.

2. DISTINCTIVE FEATURES OF YS-11

YS-11 was designed to seat 52/60 passengers and to have the cheapest direct operating cost within a short stage length of 600 n.m. The reason why we decided upon a maximum stage length of 600 n.m. is because the longest stage length in Japan, between Tokyo and Fukuoka is 550 n.m. In fact, in most countries of the world, 100 to 600 n.m. is a most common and busy flight range.

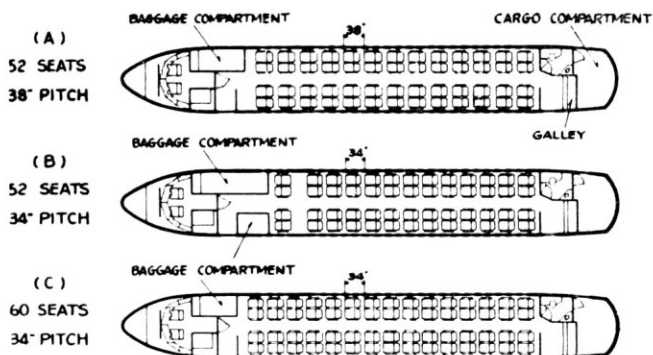


FIG. 3. Cabin arrangements.

Considered from the standpoint of stage length, it can be said that YS-11 was designed to replace DC-3. Japan, at present, is using ten DC-3s, while the total number of DC-3s used throughout the world is over 1600. It is only natural that designing of a plane which replaces a DC-3 would be a most challenging objective for transport aircraft designers. In 1956,

when we had first attempted the designing of YS-11, a modernized version of a 24/30 passenger DC-3 was contemplated as most appropriate. But, YS-11 is scheduled to go into service over a 10-year period starting from 1962. According to our research on upward trends of monthly average of domestic passengers in Japan during the years 1952-56, we estimated a monthly average of over 60,000 domestic passengers in 1962, as compared with 33,863 passengers in 1956. This, it seems, is a universal tendency in the world. Moreover, as a result of operations research mentioned later, we came to the conclusion that we can realize bigger profits by using transport planes with somewhat larger capacity than the expected number of passengers. Thus, we decided to have as many seats as our budget would allow us, namely, 52 seats in a 38-inch pitch and 60 seats in a 34-inch pitch as shown in Fig. 3. In other words, as compared to the twin turboprop transport aircraft, Friendship, Dart-Herald and Avro 748, which were manufactured in the post-war years in replacement to DC-3, YS-11 is one size larger.

Take-off and landing performance is another feature to which we attached great importance. In Japan, the standard length of a runway in a local airport is generally about 1200 meters. In summer, when the

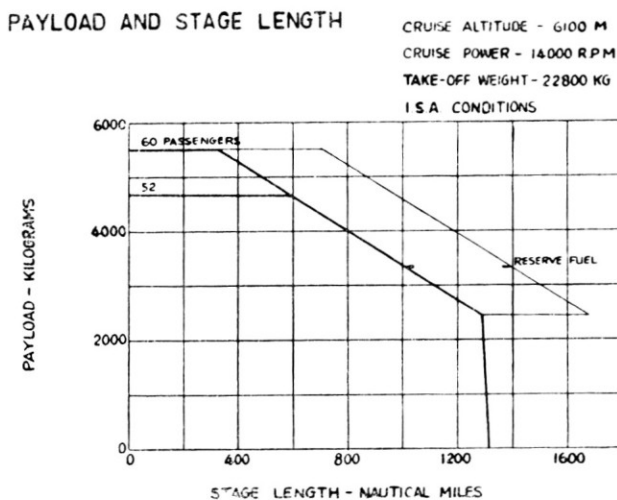


FIG. 4. Payload and stage length.

temperature is high, present transport planes must often decrease their fuel or payload for take-off. Airports with 1200-meter-long runways may be common not only in Japan but in other countries as well. YS-11 was designed not to exceed 1200 meters both in landing and take-off field length under a high temperature of $ISA+23^{\circ}C$ ($38^{\circ}C$) at sea level. This

was made possible mainly due to the superior capacity of Rolls-Royce Dart R. Da-10/1 engine, which, by means of water-methanol injection can maintain a 2750 shaft h.p. of take-off power at sea level at a wide range of $ISA \pm 30^{\circ}C$.

In order to increase take-off thrust, we used a four blade Rotol propeller with a big diameter (14 ft 6 in.), which hitherto had never been used in a Dart-Rotol combination. This propeller has a superior static thrust of 3930 kg with 2750 shaft h.p., while it can maintain a high cruise efficiency of over 86% in flight. We also used a pitch reversal system for the first time in a Dart-Rotol combination which reduced field length considerably. Specifications of YS-11 are as follows:

Powerplant: Two Rolls-Royce Dart RDa-10/1 turboprop engines, (Take-off power: 2660 ehp dry and 3060 ehp with water/methanol) Rotol four-blade propellers, 4.42 m (14 ft 6 in.) diameter.

Dimensions: Span 32.0 m (105 ft), Length 26.3 m (86 ft 3 in.), Height 9.11 m (29 ft 11 in.), Maximum width of fuselage 2.88 m (9 ft 5 in.), Wing area 94.8 m² (1,020 sq. ft), Aspect ratio 10.8.

Weights and loadings: Operating weight empty 14,000 kg (30,856 lb.), Take-off gross weight 22,800 kg (50,251 lb.), Maximum landing weight 21,800 kg (48,047 lb.), Maximum payload 5500 kg (12,122 lb.), Maximum fuel weight 5460 kg (12,034 lb.), Wing loading 241 kg/m² (49.4 lb./sq. ft), Power loading 3.72 kg/ehp (8.20 lb./ehp).

Performance: Cruising speed (flying weight 21,800 kg, Altitude 6100 m, 14,200 r.p.m.) 261 kt, Maximum stage length (with reserve fuel*, 2550 kg payload) 1280 n.m., Normal stage length (with reserve fuel*, 52 passengers) 590 n.m., Maximum payload stage length (with reserve fuel*, 60 passengers) 340 n.m., Take-off field length (ISA) 990 m (3250 ft), (ISA + 23°C) 1115 m (3660 ft), Landing field length (ISA) 1115 m (3660 ft) (ISA + 23°C) 1190 m (3900 ft).

3. THE PROCESS OF DECIDING THE BASIC FORM

In deciding the basic form of YS-11, we selected wing loading W/S , power loading W/P and aspect ratio λ with the purpose of keeping direct operating cost to the minimum, as against a given number of passenger seats n and length L . The value of wing thickness and taper ratio of wing which seems to be related to direct operating cost, has been decided for other reasons. In order to fulfil our purpose, we employed the following method.

* For 45 min. stand off at 5000 ft and 200 n.m. diversion or return.

The gross weight of plane, W , consists of the following items:

$$W = k_1W + k_2W + k_3P + k_4 + k_5n + W_f \tag{3.1}$$

where k_2 : weight of wing and empennage/gross weight

k_3 : weight of power plant/ P

k_5 : weight relating to passengers/ n

P : take-off power

n : number of passenger seats

W_f : weight of fuel

$k_1W + k_4$: weight of all other items not included above.

As a result of analyzing the weight break down of some recent transport planes, the value of these coefficients were decided as follows:

k_1, k_3, k_4 and k_5 being constant, we assumed $k_1 = 0.115, k_3 = 0.62$ kg/ $P, k_4 = 1360$ kg, $k_5 = 132$ kg/passenger.

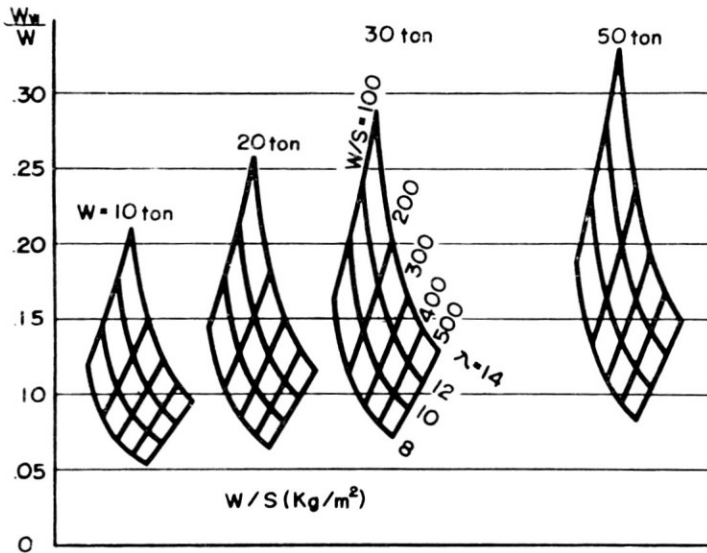


FIG. 5. Wing weight/gross weight ratio for various combinations of wing loading, aspect ratio and gross weight for a given thickness ratio and taper ratio of the wing.

As the taper ratio and thickness of wing are fixed, W_w/W , the weight of wing vs. gross weight are given as the function of $W, W/S$ and λ as shown in Fig. 5. Taking into consideration weight of empennage, k_2 is obtained by multiplying W_w/W by 1.225. W_f/W is obtained by SBAC method as follows:

$$\frac{W_f}{W} = 1.05 \left[\frac{W_{f1}}{W} + \frac{W_{f2}}{W} + \frac{W_{f3}}{W} + \frac{W_{f4}}{W} \right] \tag{3.2}$$

Where, suffixes 1, 2, 3 and 4 each represent climb, descent, cruise and stand off respectively and are obtained by the following formula.

$$\frac{W_f}{W} = c \frac{\sigma}{W/P} t \quad (3.3)$$

Where c : specific fuel consumption

σ : power at particular flight condition/take-off power

t : time of flight

By making a proper assumption of the aerodynamic characteristics, propeller efficiency, specific fuel consumption of engine, programming of cabin pressurization, and rate of cabin altitude descent, W_f/W is given as a function of W , W/S , W/P and L . In calculating cruising time, for a stage length less than 200 n.m. diversion distance was considered as equal to stage length, while for a stage length of over 200 n.m., it was considered as 200 n.m. constant, the stand off time being 45 min.

Consequently, gross weight is

$$W = \frac{k_4 + k_5 n}{1 - k_1 - k_2 - \frac{k_3}{W/P} - \frac{W_f}{W}} \quad (3.4)$$

Among various coefficients, although k_2 and W_f/W , as mentioned above, are the function of W itself, the influence of W being comparatively small, the value of W was assumed by approximation for this purpose.

The direct operating cost C yen/passenger nautical mile (360 yen = 1 U.S. dollar) was obtained by SBAC method, while the price rate or unit price was decided in accordance with the actual situation in Japan. The items of C are as follows:

$$\text{Depreciation: } C_1 = \frac{1}{V_b n} \frac{1}{y \mu} [F_a + 1.4 F_e + F_p]$$

$$\text{Insurance: } C_2 = \frac{1}{V_b n} \frac{1}{\mu} [0.034(F_a + F_e + F_p) + 30,000n + 152,500]$$

$$\text{Maintenance and overhaul cost: } C_3 = \frac{1}{V_b n} \left[0.475 W_a + \frac{F_e + F_p}{5400} \right]$$

$$\text{Taxes: } C_4 = \frac{1}{V_b n} \frac{1}{\mu} \cdot 0.012(F_a + F_e + F_p)$$

$$\text{Crew cost: } C_5 = \frac{1}{V_b n} \cdot 2700$$

$$\text{Landing cost: } C_6 = \frac{1}{nL} (0.0454W - 500)$$

$$\text{Fuel and oil cost: } C_7 = \frac{1}{nL} F_f$$

Where V_b : block speed (kt)

N : number of passenger seats

L : stage length (n.m.)

y : depreciation period = 8 years

Fa : cost of airframe and equipment = 22,000 yen/kg $\times W_a$

Fe : cost of engine = 8500 yen/hp $\times P$

Fp : cost of propeller = 1500 yen/hp $\times P$

Ff : cost of fuel per flight

μ : utilization

Taking the weight of a passenger = 85 kg, weight of engine and propeller = 0.44 kg/hp, cost of fuel 24 yen/kg in each of the above items, the following formula was obtained. t was block time.

$$C = \frac{t}{nL} \frac{1}{\mu} \left[\left\{ 1.710 \left(1 - \frac{W_f}{W} \right) + \frac{496}{W/P} \right\} W - 290,000n - 804,500 \right] + \frac{t}{nL} \left[\left\{ 0.475 \left(1 - \frac{W_f}{W} \right) + \frac{1,394}{W/P} \right\} W - 89n + 2,434 \right] + \frac{1}{nL} \left[\left\{ 0.0454 + 11.1 \frac{W_f}{W} \right\} W - 500 \right] \tag{3.5}$$

$$L = 440 \text{ n.m.} \quad n = 50$$

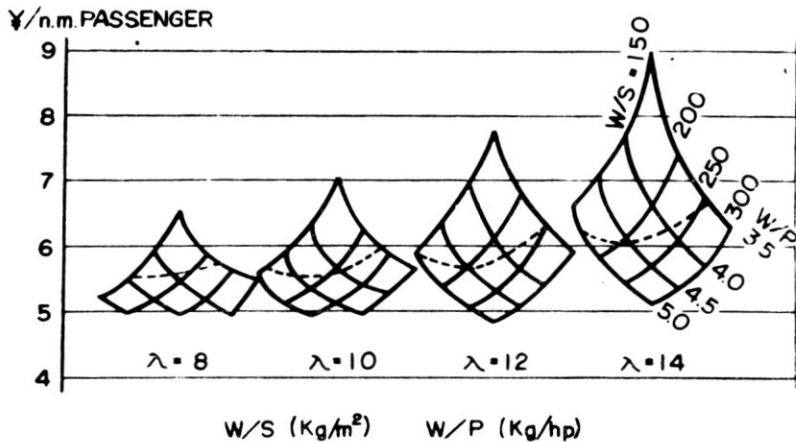


Fig. 6. Direct operating cost for various combinations of wing loading, power loading and aspect ratio of the wing.

Thus, W for a given N and L being the function of W/S , W/P and λ , the direct operating cost for a given n and L is to be considered also as the function of W/S , W/P and λ .

Consequently, a systematic calculation of the relationship between W/S , W/P , λ and direct operating cost was made as against the various

combination of number of passengers n and stage length L . Figure 6 is an example for 50 passengers and 440 nautical miles.

By making these charts, the combination of W/S , W/P and λ can be chosen in order to keep the direct operating cost as low as possible.

On the other hand, take-off field length is limited to 1200 m. There are also a number of requirements of CAR regulations as to the angles of climb. On the chart of Fig. 6, the dotted lines show the 1200-meter limit of take-off field length. It is needless to point out that the combination of W/S , W/P and λ is chosen above the limit of this line. The take-off field length requirements of a YS-11 being severe, the angle of climb requirements are covered by the former.

According to the present YS-11 plan, $\lambda = 10.8$, $W/S = 241$ kg/m², $W/P = 3.72$ kg/hp. By plotting these figures into a $\lambda = 10$ chart, it was revealed that the direct operation cost had been chosen at the lowest.

4. ESTIMATION OF NUMBER OF PASSENGERS IN DOMESTIC SCHEDULED AIR TRANSPORT

As a reference in deciding the number of seats of the YS-11, we estimated the number of passengers of domestic scheduled air transport in Japan in the years 1962-1963, when it was expected to go into service. This estimate was carried out in 1957, when research data covering the five years of domestic scheduled transport since its reopening until the end of 1956 were at hand. It studying the upward trend of these five

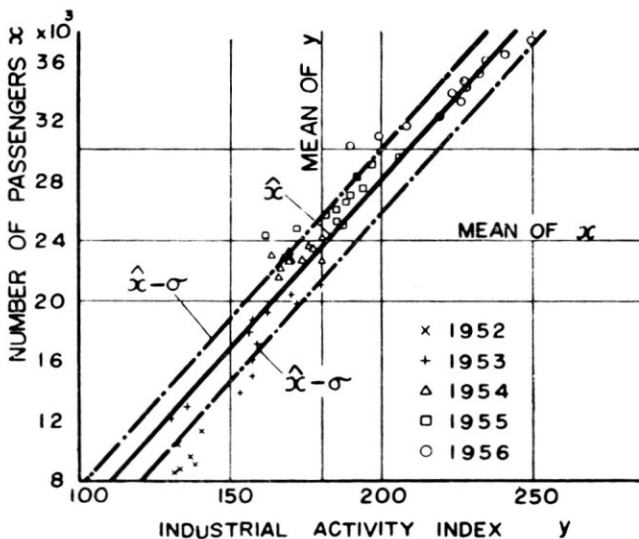


FIG. 7. Scatter diagram of number of passengers per month and industrial activity index.

years, it had been anticipated, that there existed a considerably high degree of correlation between the number of passengers and industrial activity index of Economic Investigation Board. Thus, as shown on Fig. 7, the scatter diagram of number of passengers and industrial activity index result in the following correlation coefficient:

$$\gamma = 0.96144$$

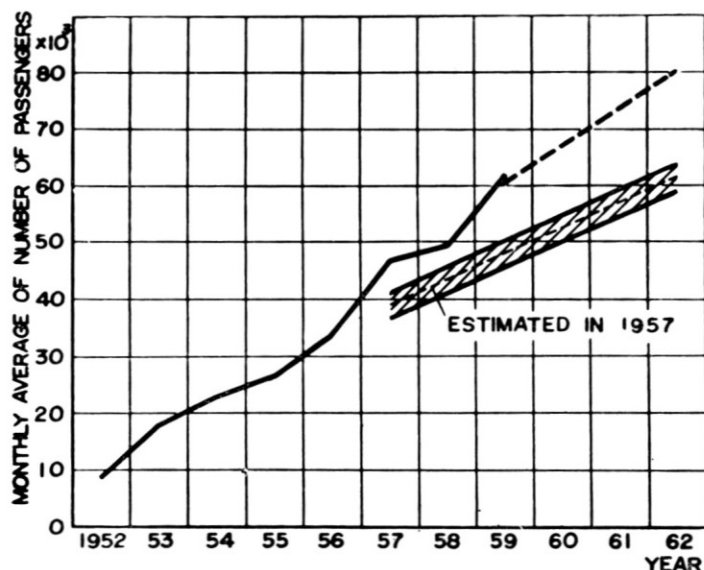


FIG. 8. Increase tendency of monthly domestic passengers in Japan.

If the number of passengers is x and the industrial activity index is y , the mean value of each, between July 1952 and December 1956 will be

$$\bar{x} = 23,375, \quad \bar{y} = 179.6$$

The standard deviation of each will be

$$\sqrt{v_x} = 7773.4, \quad \sqrt{v_y} = 30.4138$$

By using correlation coefficient γ , regression line is

$$\frac{x - \bar{x}}{\sqrt{v_x}} = \gamma \frac{y - \bar{y}}{\sqrt{v_y}} \tag{4.1}$$

Thus

$$x = -20,758 + 245.7y \tag{4.2}$$

The standard deviation of estimated value is

$$\sqrt{(1 - \gamma^2)v_x} = 2160 \tag{4.3}$$

Consequently, if the long run tendency of an estimated number of passengers is \hat{x} , the real value will be with 2/3 probability

$$\hat{x} \pm 2160$$

and with 95% probability.

$$\hat{x} \pm 4320$$

As the Economic Planning Board made public the Economic Five-year-plan, in which it designated a goal for economic activities in 1962, the future value of number of passengers were estimated by using formula 4.2, as shown by a shaded area in Fig. 8. After the above estimation had been completed, the actual data of the years 1957-59 were obtained. As shown in Fig. 8, the actual data considerably exceeds our estimated value, and from this tendency it may be natural to estimate that the average number of domestic passengers in Japan will exceed 80,000 per month in 1962.

5. OPERATIONS RESEARCH TO DETERMINE THE OPTIMUM NUMBER OF SEATS

In the basic design of a transport airplane, "how many may be the most suitable number of passenger seats for a certain air-route?" is one of the problems to be studied by operations research. The number of seats must be determined so as to make maximum profit, which is defined as the surplus of income or passenger fare over the operating cost, the latter being dependent on the number of seats.

Assuming the probability density of passengers by some mathematical model, the optimum numbers were obtained for various values of the cost-profit ratio. The method is an extension of the famous newsboy problem.

Symbols:

- | | |
|---|---|
| a : | net profit per one passenger carried (yen/passenger, flight) |
| b : | direct operating cost per hour (yen/hour) |
| c : | direct operating cost per one flight (yen/flight) |
| W_0 : | the part of gross weight independent of the number of passenger seats (kg) |
| $k_1 = \frac{\partial b}{\partial W}$: | rate of increase of direct operating cost due to the unit increase of gross weight (yen/hour, kg) |
| $k_2 = \frac{\partial b}{\partial V_b}$: | rate of increase of direct operating cost due to the unit increase of block speed (yen/hour, kt) |

- k_3 : the part of direct operating cost per hour independent of the increase of gross weight or of block speed (yen/hour)
 n : number of passenger seats
 n_c : number of crew seats
 n_* : optimum number of passenger seats
 t : block time
 x : number of passengers proposed for a flight
 W : gross weight (kg)
 L : stage length (n.m.)
 α : rate of increase of direct operating cost per one flight due to the increase of one passenger seat (yen/passenger, flight)
 β : the part of direct operating cost per one flight independent of the number of passenger seats (yen/flight)
 η : growth factor, i.e. rate of increase of gross weight due to the increase of one passenger seat (kg/passenger)
 $e(P/n)$: reduced expectation of profit (yen/flight)
 $E(Pn)$: expectation of profit of the air-line company when the number of passenger seats is n (yen/flight)
 $p(x)$: probability density function
 $P(h) = \sum_0^h p(x)$: cumulative distribution function
 $P(xn)$: net profit of the airline company when x passengers appear and n seats are prepared (yen/flight)

The profit per one flight obtained by the airline company is $ax-bt-c$ when $x \leq n$ passengers appear and $an-bt-c$ when there are $x > n$ passengers:

$$P(x/n) = \begin{cases} ax-bt-c, & x \leq n; \\ an-bt-c, & x > n. \end{cases} \quad (5.1)$$

Net profit a is the difference between the fare per one passenger and the costs of refreshments and others served for one passenger during the flight.

The direct operating cost per hour b is estimated to include the following costs: crew cost, cost of fuel and oil consumed, depreciation, and maintenance of the flight equipment. All these are accounted for one hour. Landing fee and other costs which are natural to be accounted for one flight, are all included in c .

In this paragraph the direct operating cost is assumed to be

$$b = k_1W + k_2V_b + k_3 \quad (5.2)$$

where the coefficients k_1 , k_2 and k_3 are constants statistically determined for several ranges of V_b and L .

The block time t is the ratio of the stage length L and the block speed V_b , and we have

$$t = L/V_b. \quad (5.3)$$

The gross weight may be simply assumed as

$$W = \eta n + W_0 \quad (5.4)$$

The growth factor η is defined by

$$\eta = \frac{\partial W}{\partial n} = 185 \quad (5.5)$$

We have

$$bt + c = \{k_1(\eta n + W_0) + k_2 V_b + k_3\}L/V_b + c = an + \beta \quad (5.6)$$

where

$$\alpha = \frac{k_1 \eta L}{V_b} \quad (5.7a)$$

and

$$\beta = (k_1 W_0 + k_2 V_b + k_3)L/V_b + c \quad (5.7b)$$

The a is the rate of increase of direct operating cost per one flight due to the increase of passenger seats by unity.

Hence we have

$$P(x/n) = \begin{cases} ax - dn - \beta, & x \leq n; \\ an - an - \beta, & x > n. \end{cases} \quad (5.8)$$

If $p(x)$ is the probability density of passenger's number x , then we have the expectation of profit of the airline company for one flight when the number of passenger seats is n as follows:

$$\begin{aligned} E(P/n) &= \sum_{x=0}^{\infty} P(x/n)p(x) \\ &= \sum_{x=0}^n (ax - an - \beta)p(x) + \sum_{n+1}^{\infty} (an - an - \beta)p(x) \\ &= \sum_0^n (ax - an - \beta)p(x) + (an - an - \beta) \left[1 - \sum_0^n p(x)\right] \\ &= a \sum_0^n xp(x) + an \left[1 - \sum_0^n p(x)\right] - an - \beta \\ &= a \sum_0^n xp(x) - an \sum_0^n p(x) + (a-a)n - \beta \end{aligned} \quad (5.9)$$

Optimum number of passenger seats n_* is determined as the value of n giving the maximum value of $E(P/n)$, and must satisfy the following conditions:

$$E(P/n_*) - E(P/n_* - 1) \geq 0 \tag{5.10a}$$

and

$$E(P/n_*) - E(P/n_* + 1) \geq 0 \tag{5.10b}$$

From (5.9),

$$\begin{aligned} E(P/n_*) - E(P/n_* - 1) &= aP_{(n)} - a \sum_0^n p(x) + (a - \alpha) \\ &= a \{1 - \alpha/a - P(n_* - 1)\} \end{aligned} \tag{5.11}$$

We have

$$1 - \alpha/a - P(n_* - 1) \geq 0 \tag{5.12a}$$

and similarly,

$$1 - \alpha/a - P(n_*) \leq 0 \tag{5.12b}$$

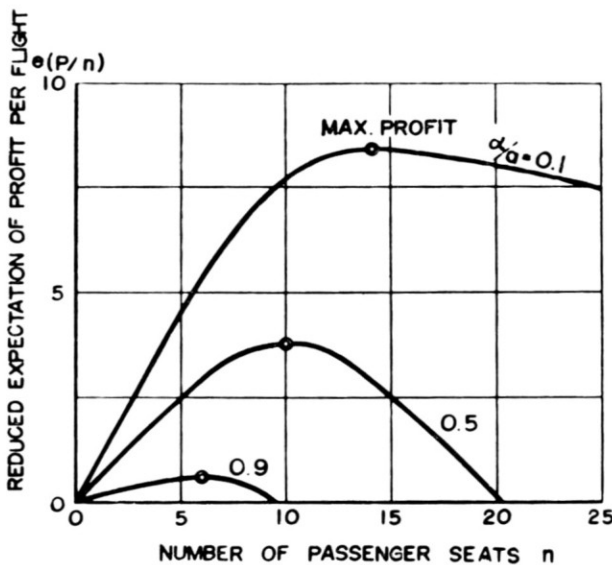


FIG. 9. Reduced expectation of profit for various values of cost-profit ratio.

Therefore, to find the optimum n_* , for the given passenger cost-fare ratio α/a , we have simply to find the value n_* from the table of cumulative distribution of passengers' number which satisfies the above two conditions, (5.12a) and (5.12b).

If the probability density law of passengers' number is a Poisson distribution

$$p_{(x)} = e^{-m} \frac{m^x}{x!}$$

then the cumulative distribution function is

$$P(h|m) = \sum_0^h p(x) = \sum_0^h e^{-m} \frac{m^x}{x!} \quad (5.13)$$

of which the numerical value is given by the statistical table. Then we have

$$\sum_0^n xp(x) = \sum_1^n e^{-m} \frac{\lambda_m^{x-1}}{(x-1)!} \cdot m = m \sum_0^{n-1} e^{-m} \frac{\lambda_m^x}{x!} = mP(n-1|m)$$

The expectation of profit is

$$E(P|n) = amP(n-1|m) - anP(n|m) + (a-\alpha)n - \beta \quad (5.14)$$

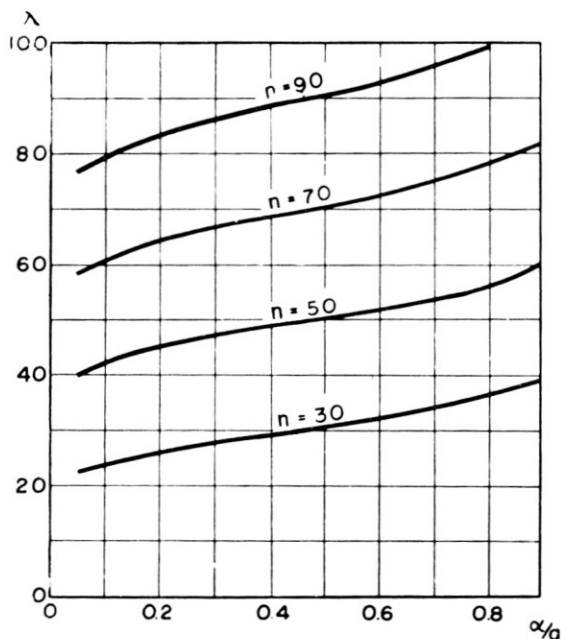


FIG. 10. The optimum number of seats for a given cost-profit ratio and mean number of passengers per flight.

The optimum number of seats n_* gives the maximum value of

$$e(P|n) = mP(n-1|m) - nP(n|m) + (1-\alpha/a)n \quad (5.15)$$

Figure 9 gives $e(P|n)$ for various values of n , when $m = 10$, $\alpha/a = 0.1$, 0.5 and 0.9 .

The curve of the reduced expectation of profit per one flight $e(P|n)$ increases rapidly and then declines slowly after it reaches the maximum point when cost-profit ratio is low, while the curve of high cost-profit ratio increases steadily and falls down quickly. It is recommended for a profitable airline with small value of a/a to use a plane having larger capacity of passenger seats while for a line with big value a/a to use smaller capacity than the average number of passengers. The risks for any unsuitable operation are shown by these curves.

The optimum values n_* for various a/a are given in Fig. 10.

The value of a/a is 0.25 approximately for YS-11. Thus we decided to have as many seats as our budget would allow as, namely 52/60 seats, though YS-11 was considered as a modernized replacement of the 24/30 passenger DC-3.

DISCUSSION

V. S. PYSHNOV: To study flying performances of passenger planes with propeller power plants we use the basic expression for the lift

$$a_{\max} = K_L (Pb)^{2/3} (\rho/\rho_0)^{1/3}$$

a_{\max} is the maximum lift in kg provided by the engine; P is the engine's power in metric h.p. b is the wing space in metres; ρ/ρ_0 is the relative air density;

$$K_L = 8.5(L/D)_{\max}^{1/3} \eta^{2/3} (K_{AK})^{1/3} \approx \sim 7.0(L/D)_{\max}^{1/3}$$

$(L/D)_{\max}$ is the maximum aerodynamic quality; η is the propeller efficiency; K_{AK} is the efficiency of aspect ratio close to 0.9 for usual wings and somewhat lower for swept back wings.

Practically for modern planes $K_L = 16-18$. From this ratio of a_{\max} and plane weight W one may find the load factor for any altitude by substituting the corresponding values of the engine power P and relative density ρ/ρ_0 .

To characterize take-off conditions we take the value $K = \frac{W}{(Pb)^{2/3}}$. At $K_W = 7-7.5$ the take off is easy; at $K_W = 8-8.5$, the take off is satisfactory; at $K_W = 10$ the take off is heavy and at $K_W = 12$ it is very heavy. For two-engine planes K_W should not exceed 7.5. Then in case of one engine failure we obtain $K_W = 12$. In case of four-engine planes K_W may be assumed to be 8-8.5 and then if one engine fails we obtain $K_W = 10.5$.

The weight of an empty plane W_0 is estimated from the value $K_{W_0} = \frac{W_0}{(Pb)^{2/3}}$.

It is usually in the limit 4-4.5. If $K_{W_0} > 5$ it means that the structure is too heavy or that the engine's power does not correspond to it. If $K_{W_0} < 4$ it may indicate the insufficient strength of the structure in using the maximum lift a_{\max} while manoeuvring.

J. R. EVANS: Professor Kimura has described a very interesting method of determining the optimum size of a transport aircraft. It would be valuable to have a comparison with other methods. One approach would be to consider all the routes required to be flown in relation to the size of aircraft which provides the best frequency for the predicted

traffic. A further alternative is to optimise the aircraft so that it is capable of performing the maximum work when meeting climb and airfield performance requirements. In the case of a short range propeller driven aircraft, this is approximately equivalent to carrying the maximum payload that the chosen powerplant will permit. It might well be found that this last approach would lead to a rather larger aircraft than the present YS-11.